Solutions

7.2 From the free electron theory [eqn 6.16], $E_F = \frac{h^2}{2m^*}\left(\frac{3N}{8\pi}\right)^{2/3}$ (note that we replace the actual mass m by the effective mass m^*).

Now N = no. of electrons available for conduction per unit volume

= density of atoms x no. of free electrons contributed by each atom

$= \left(\frac{530}{6.94} \times 6.02\times10^{26}\right) \times 1 = 4.60\times10^{28}\ m^{-3}$

and $E_F = 4.2$ eV

Hence $m^* = \frac{h^2}{2E_F}\left(\frac{3N}{8\pi}\right)^{2/3} = 1.01\times10^{-30}$ kg or $\underline{1.11\ m_o}$

7.3 From [eqn 7.30] : $E = E_1 - 2A\cos ka$, it can be deduced that the width of the band is 4A.

The effective mass $= \hbar^2 / \frac{\partial^2 E}{\partial k^2} = \frac{\hbar^2}{2Aa^2\cos ka}$

Hence, at the bottom of the band (k = 0), the effective mass is $\hbar^2/(2Aa^2)$ which is inversely proportional to the width of the band. (Q.E.D.)

7.4 The group velocity $V_g = \frac{1}{\hbar}\frac{\partial E}{\partial k} = \frac{1}{\hbar}\frac{\partial}{\partial k}\left(\frac{\hbar^2\alpha^2}{2m}\right)$ (from [eqn 7.5])

$= \frac{\hbar y}{a^2 m}\frac{\partial y}{\partial k}$ where $y = \alpha a$

Differentiate [eqn 7.3] with respect to k

7.8 Similar to Ex 7.7,

$$V_{\pm} = \pm \frac{2}{a}\left\{ \int_0^{(a-w)/2} -\frac{V_o}{2}\cos 2kx\,dx + \int_{(a-w)/2}^{a/2} \frac{V_o}{2}\cos 2kx\,dx \right\}$$

$$= \pm \frac{V_o}{2ka}\left(\sin k(a+w) - \sin k(a-w) - \frac{1}{2}\sin 2ka \right)$$

For the n^{th} energy band, $k = n\pi/a$ and as $w \longrightarrow 0$, $V_o w$ = constant,

$$V_{\pm} = \pm \lim_{w\to o} \frac{V_o}{2n\pi}\left[\sin n\pi(1 + w/a) - \sin n\pi(1 - w/a) \right]$$

$$= \pm \frac{V_o}{2n\pi}\,\frac{2n\pi w}{a}\,(-1)^n = \pm (-1)^n V_o w/a$$

$\therefore$ the total energy $= \hbar^2(n\pi/a)^2/2m \pm V_o w/a$

Thus the width of the n^{th} allowed band is

$$\left[\hbar^2(n\pi/a)^2/2m - V_o w/a\right] - \left[\hbar^2[(n-1)\pi/a]^2/2m + V_o w/a\right]$$

$$= h^2(2n-1)/8ma^2 - 2V_o w/a$$

Hence the width of the allowed band increases with n, but the width of the forbidden band remains constant.

7.9 The collision time is assumed to be the same for the hole and electron. So the one with a smaller effective mass will have a higher mobility (recall that mobility $= e\tau/m^*$).

Note that $m^* = \hbar^2 / \frac{\partial^2 E}{\partial k^2}$. Since the conduction band is higher than the valence band, the width of the former is larger. Assuming that the shape of the E-k curves are similar in both bands (e.g. $E = E_1 - 2A\cos ka$ as in the Feynman's model), then $\frac{\partial^2 E}{\partial k^2}$ would be larger in the conduction band. Hence in general, an electron will have a smaller effective mass and so a higher mobility.

... of impurity
... range T_1 to $2T_1$
... temperature determine

... of carriers in an
... carrier concentra-
... value by an amount
... zero according to the

Solutions

8.1 The Fermi level is given by [eqn 8.24] : $E_F = \frac{E_g}{2} + \frac{3}{4} kT \log\left(m_h^*/m_e^*\right)$

(for derivation, see section 8.2 of the text)

∴ the difference of the Fermi level from the middle of the gap

$$= E_F - E_g/2 = \frac{3}{4} kT \log\left(m_h^*/m_e^*\right)$$

$$= \frac{3}{4} \times 0.025 \times \log\left(\frac{0.65}{0.067}\right) = \underline{0.043\ \text{eV}}$$

(5) USING THE KRONIG-PENNEY MODEL SHOW THAT FOR $P \ll 1$, THE ENERGY OF THE LOWEST BAND IS $E = \frac{\hbar^2 P}{ma^2}$

eqn 7.3: $\cos(ka) = P\frac{\sin(\alpha a)}{\alpha a} + \cos(\alpha a)$

FOR LOWEST ENERGY, $ka \ll 1 \Rightarrow \cos(ka) \simeq 1$

ALSO, THE LOWEST ENERGY IS FOUND NEAR $\alpha a \simeq \pi/4$, WHERE $\cos(\alpha a)$ AND $\sin(\alpha a)$ ARE FAIRLY LINEAR AND CAN BE APPROXIMATED BY THE FIRST COUPLE TERMS OF THEIR TAYLOR SERIES:

$\cos(\alpha a) \simeq 1 - \frac{(\alpha a)^2}{2}$, $\sin(\alpha a) \simeq \alpha a$

$\Rightarrow 1 \simeq P + \left(1 - \frac{(\alpha a)^2}{2}\right) \Rightarrow P \simeq \frac{(\alpha a)^2}{2} \Rightarrow$

$P = \left(\frac{2mE}{\hbar^2}\right) \cdot \frac{a^2}{2} \Rightarrow \underline{E = \frac{\hbar^2 P}{ma^2}}$