## Solutions

7.2 From the free electron theory [eqn 6.16],  $E_F = \frac{h^2}{2m^*} \left(\frac{3N}{8\pi}\right)^{2/3}$  (note

that we replace the actual mass m by the effective mass  $m^*$ ).

Now N = no. of electrons available for conduction per unit volume

= density of atoms  $\boldsymbol{x}$  no. of free electrons contributed by each atom

$$= \left(\frac{530}{6.94} \times 6.02 \times 10^{26}\right) \times 1 = 4.60 \times 10^{28} \text{ m}^{-3}$$

and  $E_F = 4.2 \text{ eV}$ 

Hence 
$$m^* = \frac{h^2}{2E_F} \left( \frac{3N}{8\pi} \right)^{2/3} = 1.01 \times 10^{-30} \text{ kg or } 1.11 \text{ m}_0$$

1 is 6.94 and its density is nass at the bottom of the

the basis of their band

he border lines of these

nduction band of lithium illed energy level in the Fermi level what average

electron theory? Assume

ity of the electron is zero

ass is a tensor whose w would the classical

of the conduction band

termine with the aid of tke w = a/2.

7.2, that w and  $V_0$  in product  $V_0 w$  is kept the widths of the nth ide that the higher the

ace there is hardly any e band is, the wider it ne for the valence and mobility, an electron

7.3 From [eqn 7.30] :  $E = E_1 - 2A \cos ka$  , it can be deduced that the width of the band is 4A.

The effective mass =  $\hbar^2 / \frac{a^2 E}{ak^2} = \frac{\hbar^2}{2Aa^2 \cos ka}$ 

Hence, at the bottom of the band (k = 0), the effective mass is  $\hbar^2/(2\mathrm{Aa}^2)$ which is inversely proportional to the width of the band.

7.4 The group velocity 
$$V_g = \frac{1}{\hbar} \frac{\partial E}{\partial k} = \frac{1}{\hbar} \frac{\partial}{\partial k} \left( \frac{\hbar^2 \alpha^2}{2m} \right)$$
 (from [eqn 7.5])
$$= \frac{\hbar y}{a^2 m} \frac{\partial y}{\partial k} \quad \text{where } y = \alpha a$$

Differentiate [eqn 7.3] with respect to k

7.8 Similar to Ex 7.7,

$$V_{\pm} = \pm \frac{2}{a} \left\{ \int_{0}^{(a-w)/2} - \frac{V_{o}}{2} \cos 2kx \, dx + \int_{(a-w)/2}^{V_{o}} \frac{V_{o}}{2} \cos 2kx \, dx \right\}$$

$$= \pm \frac{V_{o}}{2ka} \left( \sin k(a + w) - \sin k(a - w) - \frac{1}{2} \sin 2ka \right)$$

For the n  $^{\rm th}$  energy band, k = n  $\pi/a$  and as w  $\longrightarrow$  0,  $V_0^{\rm W}$  = constant,

$$V_{\pm} = \pm \frac{\lim_{w \to 0} \frac{V_{o}}{2n\pi} \left[ \sin n\pi (1 + w/a) - \sin n\pi (1 - w/a) \right]$$
$$= \pm \frac{V_{o}}{2n\pi} \frac{2n\pi w}{a} (-1)^{n} = \pm (-1)^{n} V_{o} w/a$$

... the total energy =  $\hbar^2 (n\pi/a)^2 / 2m \pm V_0 w/a$ 

Thus the width of the  $n^{\mbox{\,$t$}\mbox{\,$h$}}$  allowed band is

$$\left[ \hbar^2 (n\pi/a)^2 / 2m - V_o w/a \right] - \left[ \hbar^2 [(n-1)\pi/a]^2 / 2m + V_o w/a \right]$$

$$= h^2 (2n-1) / 8ma^2 - 2V_o w/a$$

Hence the width of the allowed band increases with n, but the width of the forbidden band remains constant.

7.9 The collision time is assumed to be the same for the hole and electron. So the one with a smaller effective mass will have a higher mobility (recall that mobility =  $e\tau/m^*$ ).

Note that  $m^* = \hbar^2 / \frac{\partial^2 E}{\partial k^2}$ . Since the conduction band is higher than the valence band, the width of the former is larger. Assuming that the shape of the E-k curves are similar in both bands (e.g.  $E = E_1 - 2A \cos ka$  as

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band. He

x dx

1 2ka )

w = constant,

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V<sub>o</sub>w/a ]

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d is higher than the uming that the shape  $= E_1 - 2A \cos ka$  as

in the Feynman's model), then  $\frac{\partial^2 E}{\partial k^2}$  would be larger in the conduction band. Hence in general, an electron will have a smaller effective mass and so a higher mobility.

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## Solutions

8.1 The Fermi level is given by [eqn 8.24]:  $E_F = \frac{E_g}{2} + \frac{3}{4} kT \log \left( \frac{m_h^*}{m_e^*} \right)$  (for derivation, see section 8.2 of the text)

, the difference of the Fermi level from the middle of the gap  $=E_F-E_g/2=\frac{3}{4}~kT~\log\left(m_h^*/m_e^*\right)$ 

$$=\frac{3}{4} \times 0.025 \times \log \left(\frac{0.65}{0.067}\right) = \frac{0.043 \text{ eV}}{0.043 \text{ eV}}$$

SUSING THE KRONIG-PENNEY MODEL SHOW
THAT FOR PELL, THE ENERGY OF THE COLEST

BAND IS E= HZP mgz

egn 7.3: cos (Ka)= P s/N(xa) + cos(xa)

FOR LOWGET ENERGY, Ka << 1 => cos (kg) = 1

ALSO, THE LOWEST ENERBY IS TOUND NEAR HAS THE MUERE COSED, AND SINDA, ARE FAIRLY UNEAR AND CAN BE APPROXIMATED BY THE FIRST COUPLE TERMS OF THEIR TAYLOR SERIES:

(05(2a) = 1- 2, SIN(2a) = 2a

=> 1=P+/1- 40/2 => P= (40)2=>

P = (2mE) · 92 => E = ti2P to2) 2 = ma2